2.39 CONDUCTORS IN ELECTRIC FIELD

Conductors

Definition 1 Conductors are materials which have very low resistance. Examples: Copper, Silver and Aluminium.

Definition 2 Conductors are materials for which no forbidden gap exists between valance band and conduction band.

Definition 3 A material is defined as a conductor if $\frac{\sigma}{\omega\in} >> 1$.

Conductors have a variety of applications in all fields of life.

2.40 PROPERTIES OF CONDUCTORS

1. Charge density is zero within a conductor.
2. The surface charge density resides on the exterior surface of a conductor.
3. In static conductors, current flow is zero.
4. Electric field is zero within a conductor.
5. Conductivity is very large.
6. Resistivity is small.
7. Magnetic field is zero inside a conductor.
8. Good conductors reflect electric and magnetic fields completely.

9. A conductor consists of a large number of free electrons which constitute conduction current with the application of an electric field.
10. A conductor is an equipotential body.
11. The potential is same everywhere in the conductor.
12. $\mathbf{E} = -\nabla V = 0$ in a conductor.
13. In a perfect conductor, conductivity is infinity.
14. When an external field is applied to a conductor, the positive charges move in the direction **E** and the negative charges move in the opposite direction. This happens very quickly.
15. Free charges are confined to the surface of the conductor and hence surface charge density, J_s is induced. These charges create internal induced electric field. This field cancels the external field.

It is interesting to note that copper and silver are not super conductors but aluminium is a superconductor for temperature below 1.14 K.

2.41 ELECTRIC CURRENT

The current through a given medium is defined as charge passing through the medium per unit time. It is a scalar, that is,

$$I \equiv \frac{dQ}{dt}, \text{Ampere}$$

Current is of three types.

1. Convection current
2. Conduction current
3. Displacement current

1. **Convection current** It is defined as the current produced by a beam of electrons flowing through an insulating medium. This does not obey Ohm's law. For example, current through a vacuum, liquid and so on is convection current.
2. **Conduction current** It is defined as the current produced due to flow of electrons in a conductor. This obeys Ohm's law. For example, current in a conductor like copper is conduction current.
3. **Displacement current** It is defined as the current which flows as a result of time-varying electric field in a dielectric material. For example, current through a capacitor when a time-varying voltage is applied is displacement current.

2.42 CURRENT DENSITIES

In electromagnetic field theory, it is of interest to describe the events at a point instead of in a large region. This is the reason why current densities are considered. Current densities are vector quantities.

Current Density is defined as the current at a given point through a unit normal area at that point. It is a vector and it has the unit of Ampere/ m^2. It is represented by **J**.

Current densities are of three types:

1. Convection current density
2. Conduction current density
3. Displacement current density

1. **Convection current density (A/m^2)** It is defined as the convection current at a given point through a unit normal area at that point, that is,

Convection current density

$$\equiv \frac{dI}{d\mathbf{S}}$$

$$\equiv \frac{dI}{dS}\mathbf{a}_n$$

where dI = differential convection current

$d\mathbf{S}$ = differential area

$= dS\mathbf{a}_n$

$\mathbf{a}_n$ = outward unit normal to dS

As convection current density is confined to specific media, it is not of much interest in this book.

2. **Conduction current density, J_c (A/m^2)** It is defined as the conduction current at a given point through a unit normal area at that point,

that is,

$$\mathbf{J}_c = \sigma\mathbf{E}$$

and

$$\mathbf{J}_c = \frac{dI}{dS}\mathbf{a}_n$$

Conduction current density exists in the case of conductors when an electric field is applied.

3. **Displacement current density, J_d (A/m²)** It is defined as the rate of displacement electric flux density with time, that is,

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

If I_d is the displacement current in a dielectric due to applied electric field, displacement current density is defined as

$$\mathbf{J}_d = \frac{dI_d}{dS}\mathbf{a}_n$$

As

$$\mathbf{D} = \in \mathbf{E}$$

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} = \in \frac{\partial \mathbf{E}}{\partial t}$$

In fact, displacement current density exists due to displacement of bound charges in a dielectric by the applied electric field.

2.43 EQUATION OF CONTINUITY

Equation of continuity in integral form is $I = \int \mathbf{J}.ds$

I = outward flow of current (A)

J = conduction current density (A/m²)

$$\nabla.\mathbf{J} = -\dot{\rho}_\upsilon$$

where

$$\dot{\rho}_\upsilon = \frac{\partial \rho_\upsilon}{\partial t}$$

Proof If Q_i is the charge inside a closed surface, the rate of decrease of charge due to the outward flow of current is given by $\left(-\frac{dQ_i}{dt}\right)$.

From the principle of conservation of charge, we have

$$I = -\frac{dQ_i}{dt} = \oint_s \mathbf{J}.ds$$

From divergence theorem, we have

$$\oint_s \mathbf{J}.ds = \int_\upsilon \nabla.\mathbf{J}\,d\upsilon$$

So,
$$\int_{v} \nabla . J dv = -\frac{dQ_i}{dt}$$

$$= -\frac{d}{dt}\int_{v} \rho_v \, d_v = -\int_{v} \dot{\rho}_v \, d_v$$

Two volume integrals are equal if the integrands are equal. So,

$$\boxed{\nabla . J = -\frac{\partial \rho_v}{\partial t} = -\dot{\rho}_v}$$ Hence proved.

In the above equation the derivative became a partial derivative as the surface is kept constant.

2.44 RELAXATION TIME (T_r)

It is also called **rearrangement time.**

Relaxation time is defined as the time taken by a charge placed in a material to reach 36.8 per cent of its initial value. It is given by

$$T_r = \frac{\epsilon}{\sigma}, \text{ sec}$$

where $\epsilon =$ permittivity (F/m), $\sigma =$ conductivity (mho/m).

Problem 2.51 Find the relaxation time of sea water whose $\epsilon_r = 81$ and $\sigma = 5$ mho/m.

Solution Relaxation time of sea water

$$T_r = \frac{\epsilon}{\sigma} = \frac{81 \times 8.854 \times 10^{-12}}{5}$$
$$= 143.37 \text{ picosecond}$$

Problem 2.52 Find the relaxation time of porcelain whose $\sigma = 10^{-10}$ mho/m, $\epsilon_r = 6$.

Solution Relaxation time of porcelain

$$T_r = \frac{\epsilon}{\sigma} = \frac{6 \times 8.854 \times 10^{-12}}{10^{-10}}$$
$$= 53.12 \times 10^{-2}$$
$$\boxed{T_r = 531.2 \text{ m sec}}$$

2.45 RELATION BETWEEN CURRENT DENSITY AND VOLUME CHARGE DENSITY

$$\boxed{\mathbf{J} = \rho_\upsilon \mathbf{V}}$$

$\mathbf{J}$ = conduction current density, A/m^2

$\mathbf{V}$ = velocity of the charge (m/s)

Proof We know that $I = \int \mathbf{J}.d\mathbf{s} = \frac{dQ}{dt}$

Consider an element charge (Fig. 2.24)

$$\Delta Q = \rho_\upsilon \Delta \upsilon = \rho \Delta s \, \Delta x$$

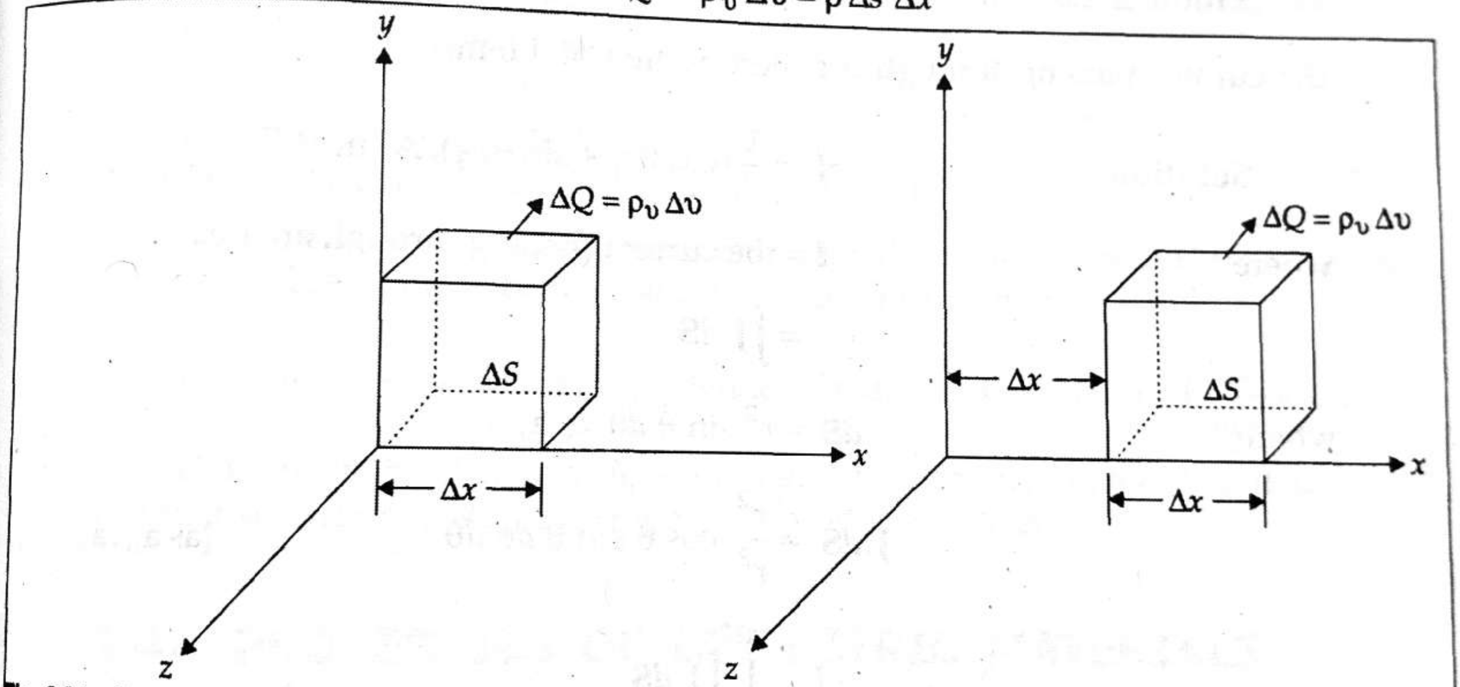


Fig. 2.24 Volume charge in motion

Assume that the charge element is oriented parallel to the coordinate axes. Let there be only an x-component of velocity. It moves a distance of Δx in a time Δt as in the figure. Therefore,

$$\Delta Q = \rho_\upsilon \, \Delta s \, \Delta x$$

The resultant current is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_\upsilon \, \Delta s \frac{\Delta x}{\Delta t} = \rho_\upsilon \Delta s V_x \qquad \left[\text{as } V_x = \frac{\Delta x}{\Delta t}\right]$$

where $V_x = x$-component of velocity of the charge.

$$\frac{\Delta I}{\Delta s} = \rho_\upsilon V_x$$

$$J_x = \rho_v V_x \,(\text{A/m}^2)$$

Similarly, if the charge moves in y and z-directions, we get

$$J_y = \rho_v V_y$$

$$J_z = \rho_v V_z$$

$$\mathbf{J} = \rho_v (V_x \mathbf{a}_x + V_y \mathbf{a}_y + V_z \mathbf{a}_z)$$

As $$\mathbf{V} = V_x \mathbf{a}_x + V_y \mathbf{a}_y + V_z \mathbf{a}_z$$

$$\boxed{\mathbf{J} = \rho_v \mathbf{V}} \quad \text{Hence proved.}$$

Problem 2.53 If the current density, $\mathbf{J} = \frac{1}{r^2}(\cos\theta a_r + \sin\theta a_\theta)$, A/m^2, find the current passing through a sphere radius of 1.0 m.

Solution $$\mathbf{J} = \frac{1}{r^2}(\cos\theta a_r + \sin\theta a_\theta), \text{ A/m}^2$$

where I = the current passing through an area

$$= \int \mathbf{J}.d\mathbf{S}$$

where $$d\mathbf{S} = r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r$$

$$\mathbf{J}.d\mathbf{S} = \frac{r^2}{r^2} \cos\theta \sin\theta \, d\phi \, d\theta \qquad [\text{as } \mathbf{a}_\theta . \mathbf{a}_r = 0]$$

$$I = \int_0^{\pi} \int_0^{2\pi} \mathbf{J}.d\mathbf{S}$$

$$I = \int_0^{\pi} \int_0^{2\pi} \cos\theta \sin\theta \, d\phi \, d\theta$$

that is, $$I = 2\pi \int_0^{\pi} \sin\theta \, d(\sin\theta)$$

Solution Electric field in free space,

$$\mathbf{E} = 6\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z, \text{ V/m}$$

$$\epsilon_0 = 8.854 \times 10^{-12}$$

The electric flux density,

$$\mathbf{D} = \epsilon_0 \mathbf{E}$$

$$= 8.854 \times 10^{-12} (6\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$\boxed{\mathbf{D} = (53.12a_x - 17.7a_y + 26.55a_z) \text{ PC/m}^2}$$

2.46 DIELECTRIC MATERIALS IN ELECTRIC FIELD

Definition 1 An ideal dielectric material is one which does not contain free electrons.

Definition 2 An ideal dielectric material is one in which the charges are well bounded and cannot be set in motion easily.

Definition 3 An ideal dielectric material is one for which there exists a large forbidden gap between valance band and conduction band.

Definition 4 A material is defined as dielectric material if $\frac{\sigma}{\omega\epsilon} << 1$.

Definition 5 A material is defined as dielectric material if it does not conduct electric current and opposes the flow of current.

2.47 PROPERTIES OF DIELECTRIC MATERIALS

1. Conductivity is zero.
2. Volume charge density, $\rho_v = 0$.
3. Electric and magnetic fields exist in a dielectric material.
4. Resistivity is ∞.
5. Electric and magnetic fields penetrate the dielectric material freely.
6. There exists no free electrons.

Dielectrics in Electric Field

An atom of a dielectric consists of a nucleus and a bunch of electrons. Similarly, a molecule of a dielectric consists of nuclei and a set of electron bunches. The charge of the nucleus is positive and the charge of the electron bunch is negative.

The atoms and molecules are electrically neutral as they contain an equal number of negative and positive charges.

Dielectrics are classified into polar and non-polar type of materials.

Polar type of dielectrics

The centres of positive and negative charges of a molecule of polar type of dielectric material are separated by a small distance. Each pair acts as a dipole and there exists a dipole moment. However, such pairs are randomly distributed in a dielectric material. Hence the overall dipole moment is zero.

If such a material is kept in an electric field, all the positive charges move in the direction of the electric field and all the negative charges move in the opposite direction. As a result, dipole moment is induced by the electric field. Under these conditions, the material is said to be under a state of polarisation.

A polar type of molecule is shown in Fig. 2.25.

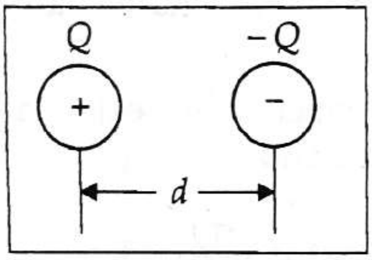


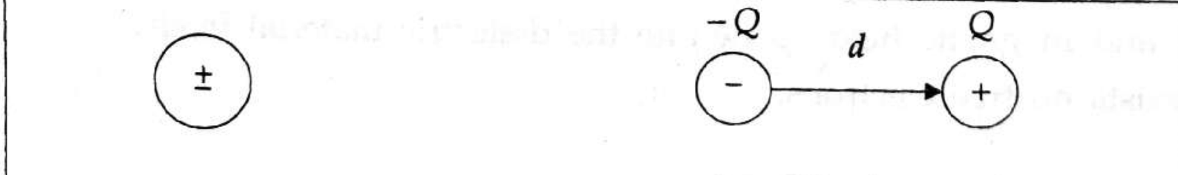
Fig. 2.25 Polar type of molecule

Examples of polar dielectrics are water, hydrochloric acid, sulphur dioxide and others.

Non-polar type of dielectrics

The centres of positive and negative charges of a molecule of non-polar type of dielectric material coincide as in Fig. 2.26, that is, there is no separation between them and hence dipole moment is zero.

However, when such a material is placed in an electric field, the centres of positive and negative charges are displaced and there exists a distance between them, that is, dipole moment is induced. Under these conditions, the material is said to be under a state of polarisation.



(*a*) Non-polar molecule (*b*) Non-polar molecule in electric field **Fig. 2.26**

Examples of non-polar dielectrics are oxygen, hydrogen, nitrogen and so on.

The important conclusion is that dielectric materials are not polarised in the absence of electric field and they are polarised in the presence of an electric field.

As a result, the electric flux density is greater than that in free space conditions with the same field intensity. The intensity of polarisation is described in terms of dipole moment and polarisation.

2.48 DIPOLE MOMENT, p

It is defined as the product of charge and distance between the centres of (+)ve and (–)ve charges of a molecule,

that is, $\mathbf{p} \equiv Q\mathbf{d}$, (C-m)

where $\mathbf{p}$ = dipole moment

Q = charge magnitude

$\mathbf{d}$ = distance vector from (–)ve to (+)ve charges of the dipole

If there are N dipoles in a dielectric material of volume $\Delta\upsilon$, the total dipole moment with the application of electric field is

$$\mathbf{p}_t \equiv \sum_{i=1}^{N} Q_i \mathbf{d}_i$$

2.49 POLARISATION, P

Polarisation is defined as the dipole moment per unit volume of the dielectric,

that is,

$$\mathbf{P} = \underset{\Delta\upsilon\to 0}{\text{Lt}} \sum_{i=1}^{N} \frac{Q_i \mathbf{d}_i}{\Delta\upsilon}$$

In some dielectrics, the polarisation, $\mathbf{P}$ is defined as

$$\mathbf{P} \equiv \chi_e \epsilon_0 \mathbf{E}$$

where χ_e = electric susceptibility of the dielectric

The charges are bound in a dielectric and the total positive bound charge on a surface, S enclosing the dielectric is given by

$$Q_b = -\int \mathbf{P}.d\mathbf{S}$$

On the other hand, the charge that remains inside the surface, S is $-Q_b$ and it is given by

$$Q_b = -\oint_{\upsilon} \nabla . \mathbf{P} d\upsilon = \oint_{\upsilon} \rho_b d\upsilon$$

If there is some free charge in the dielectric, the free charge volume charge density is ρ_υ. If ρ_b is the bound volume charge density, then the total volume charge density is given by

$$\rho_t = \rho_\upsilon + \rho_b$$

$$= \nabla . \in \mathbf{E}$$

or,

$$\rho_\upsilon = \rho_t - \rho_b$$

$$= \nabla . \in_0 \mathbf{E} + \nabla . \mathbf{P}$$

$$= \nabla . \mathbf{D}$$

$$\boxed{\mathbf{D} = \in_0 \mathbf{E} + \mathbf{P}}$$

As $\mathbf{P} \equiv \chi_e \in_0 \mathbf{E}$, we have

$$\mathbf{D} = \in_0 \mathbf{E} + \chi_e \in_0 \mathbf{E}$$

$$= \in_0 \mathbf{E} (1 + \chi_e)$$

$$\mathbf{D} = \in_0 \in_r \mathbf{E} = \in \mathbf{E}$$

where

$$\in_r = 1 + \chi_e = \frac{\in}{\in_0}$$

or,

$$\boxed{\chi_e = \in_r - 1}$$

In summary, we have

$$Q_b = \int_{\upsilon} \rho_b \, d\upsilon$$

$$Q = \int_{\upsilon} \rho_\upsilon \, d\upsilon$$

$$Q_t = \int_{\upsilon} \rho_t \, d\upsilon$$

$$\nabla . \mathbf{P} = -\rho_b$$

$$\nabla . \in_0 \mathbf{E} = \rho_t$$

$$\nabla . \mathbf{D} = \rho_\upsilon$$

Problem 2.55 A pair of negative and positive charges of $10 \mu C$ each are separated by a distance of 0.1 m along the x-axis. Find the dipole moment.